

Work, Power & Energy.

- Work - It is a scalar product of force and displacement.

$$W = \vec{F} \cdot \vec{S} = |F| |S| \cos \theta$$

Unit = Nm $\rightarrow$ Joule.

- When work done is positive then the system gets energy.
- When work done is negative then the system loses energy.

* For variable force

$$W = \int_{x_1}^{x_2} F dx$$

* The area of force & displacement gives work done in graph

* It is not necessary that if displacement is zero, total work done is also zero.

* For forces in multiple directions

$$W = \int \mathbf{F} d\mathbf{s}$$

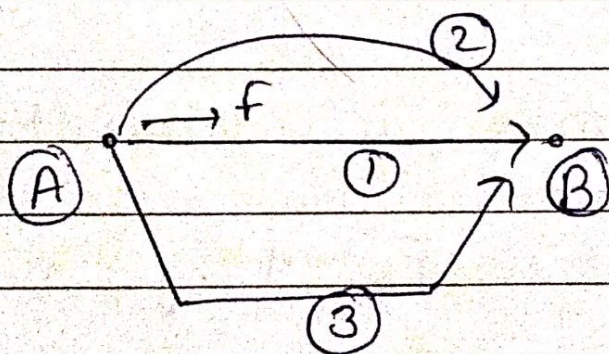
$$\text{where } d\mathbf{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}.$$

* Conservative force

• The work done by such force

1. does not depend upon ~~the~~ path

2. Depends only & only on initial & final position.



$$W_1 = W_2 = W_3$$

* Ex - gravitational force, spring force, electrostatic force.

* Work done by conservative force in a closed path is always zero.

* Non conservative forces are forces which are not conservative.

* Conservative force	Non Conservative force
i) $W_I = W_{II}$	$W_I \neq W_{II}$
• does <u>not</u> depend upon path	• does depend on path
• Closed path $W = 0$	Closed path $W \neq 0$

(*) If a Force is conservative

then $F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

Check i) $\frac{\delta F_x}{\delta y} = \frac{\delta F_y}{\delta x}$

ii) $\frac{\delta F_x}{\delta z} = \frac{\delta F_z}{\delta x}$

iii) $\frac{\delta F_y}{\delta z} = \frac{\delta F_z}{\delta y}$



Q. Find whether given force $x\hat{i} + y\hat{j}$ is conservative or non-conservative.

$$(1) \frac{dF_x}{dy} = \frac{dF_y}{dx} \Rightarrow 0 = 0$$

[F_x will be constant w.r.t y]
[F_y will be constant w.r.t x]

$$(2) \frac{dF_y}{dz} = \frac{dF_z}{dy} \Rightarrow 0 = 0$$

$$(3) \frac{dF_x}{dz} = \frac{dF_z}{dx} \Rightarrow 0 = 0$$

$\Rightarrow$ Given force is conservative.

* Work Energy Theorem.

- Kinetic energy comes into action when body is in motion.
- It depends upon frame of reference.

• work energy theorem.

⇒ Work done by all the forces on a body = Change in k.e. of the body.

→ internal + external force.

$$W_{\text{by all force}} = K.E._f - K.E._i$$

$$dW_{\text{all forces}} = d(K.E.)$$

* Potential Energy

$\Delta U = -$ Work done by conservative force.

$$\Delta U = -W_c$$

$$dU = -dW_c$$

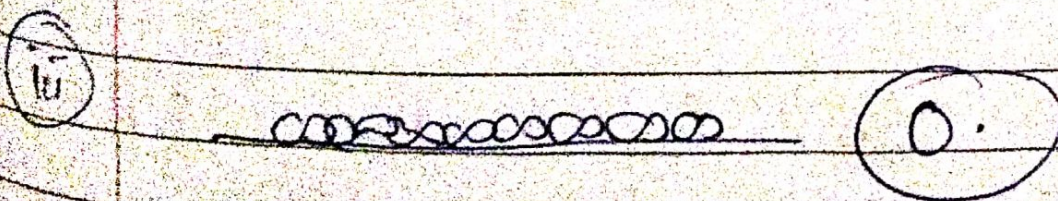
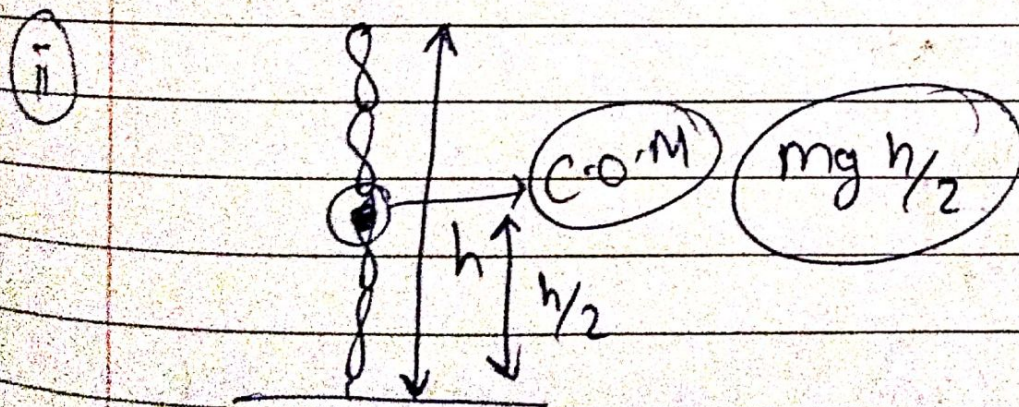
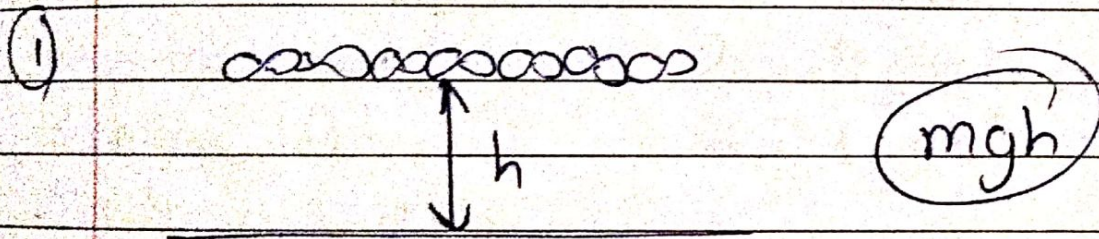
* Potential energy is not defined for non-conservative force.

$$\int du = - \int dw_c$$

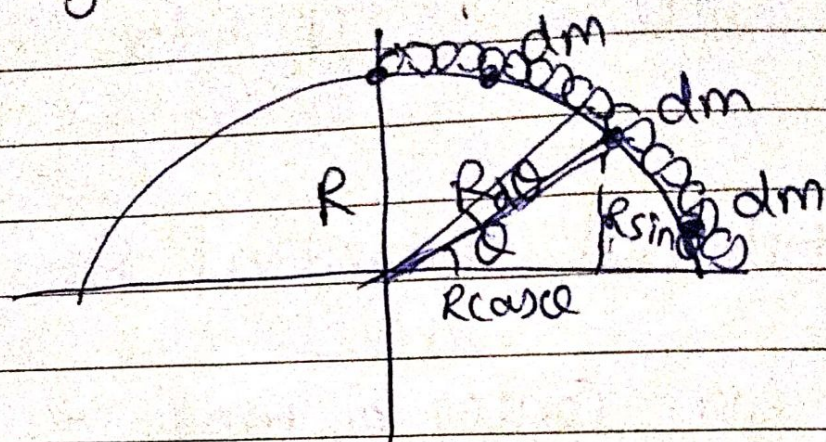
$$\int_{u_i}^{u_f} du = - \int \mathbf{F}_c \cdot d\mathbf{x}$$

* Gravitational Potential Energy. (mgh)

• Check for Center of mass of for h



* To find Potential Energy of given arrangement of chain.

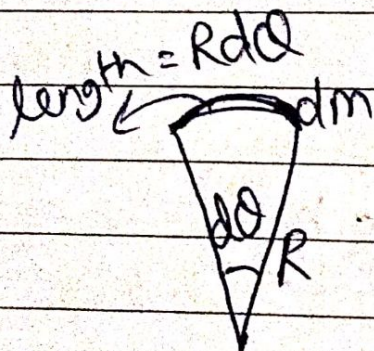


$$dU = dmgh$$

$$\Rightarrow dU = dm g R \sin \theta$$

$$\Rightarrow \int dU = \int dm g R \sin \theta$$

[Here variable is ' θ ' not dm , hence make θ variable]



Mass (m) for length $\frac{\pi R}{2}$

then mass dm for length $R d\theta$

$$\Rightarrow \frac{m R d\theta}{\frac{\pi R}{2}} = ?$$

$$dm \Rightarrow \frac{2m d\theta}{\pi}$$

* Potential Energy is invisible form of energy
it is imparted to the system, that energy
is stored as potential energy.

$$\int dU = \int_0^{\pi/2} \frac{2m}{\pi} d\theta g R \sin \theta$$

$$= \frac{2m}{\pi} g R \left[\sin \theta \right]_0^{\pi/2}$$

$$\Rightarrow \frac{2m}{\pi} g R \left[-\cos \theta \right]_0^{\pi/2}$$

$$\Rightarrow \frac{2m}{\pi} g R \left[-\cos 90 - (-\cos 0) \right]$$

$$\Rightarrow \boxed{\frac{2mgR}{\pi}}$$

* Spring Potential Energy.

$$U = \frac{1}{2} K x^2$$

• Work done by spring force

$$= \frac{1}{2} K (x_1^2 - x_2^2)$$

* Potential Energy of Spring is zero at its natural length.

$$(*) \quad U_2 - U_1 = \frac{1}{2} K x_2^2 - \frac{1}{2} K x_1^2$$

- Either you have stretched x , or compressed x ~~far~~ from natural length the potential energy stored will always be equal to $\left(\frac{1}{2} K x^2\right)$.

• Potential Energy of spring is always positive.

- Gravitational & Electrostatic potential energy can be negative.

$$(*) \quad F_x = -\frac{du}{dx}$$

$$(*) \quad F_y = -\frac{du}{dy}$$

$$(*) \quad F_z = -\frac{du}{dz}$$

③ slope of U vs x graph is Conservative force.

* Equilibrium.

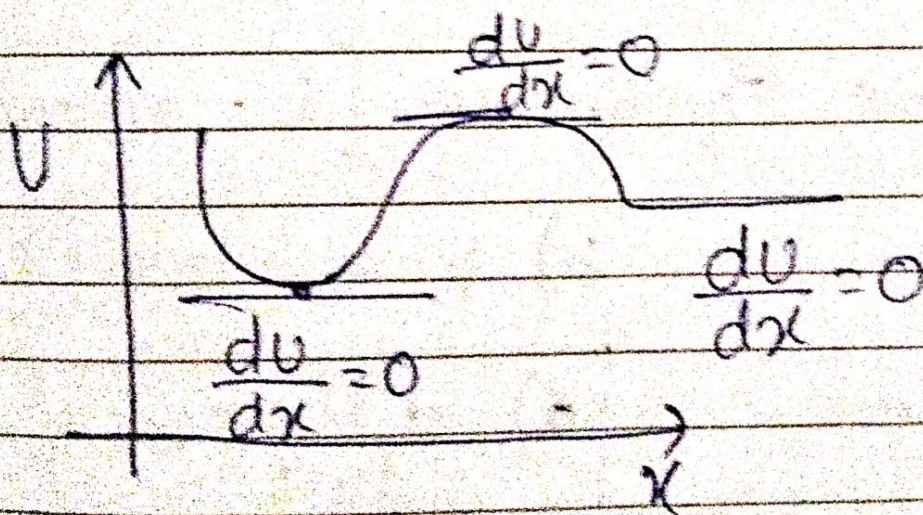
• If total force acting on body is zero, then body is in Equilibrium.

↓
Stable

↓
Unstable

↓
Neutral

④ $F_x = -ve$ (slope U v/s x)



⑤ Stable Equilibrium $\Rightarrow U \rightarrow$ minimum

⑥ Unstable Equilibrium $\Rightarrow U \rightarrow$ maximum

⑦ Neutral Equilibrium $\Rightarrow U \rightarrow$ constant.

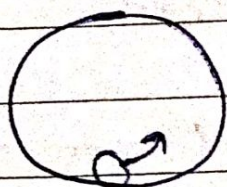
1. Stable Equilibrium [$U = \text{minimum}$]

a.) $F_{\text{net}} = 0$

b.) $U-x$ slope $= 0$

c.) $\frac{dU}{dx} = 0$

d.) $\frac{d^2U}{dx^2} > 0$



If we displace the body slightly it, returns back to stable equilibrium.

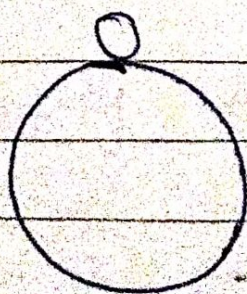
2. Unstable Equilibrium [$U = \text{maximum}$]

a.) $F_{\text{net}} = 0$

b.) $U-x$ slope $= 0$

c.) $\frac{dU}{dx} = 0$

d.) $\frac{d^2U}{dx^2} < 0$



If we displace the body slightly it will move towards ~~stable~~ equilibrium.
Stable

③ Neutral Equilibrium [U-constant]

a) $F_{\text{net}} = 0$

b) $U-x$ Slope $= 0$

c) $\frac{dU}{dx} = 0$

d) $\frac{d^2U}{dx^2} = 0$



will remain in again in neutral equilibrium position [new position].

* Conservation of Mechanical Energy

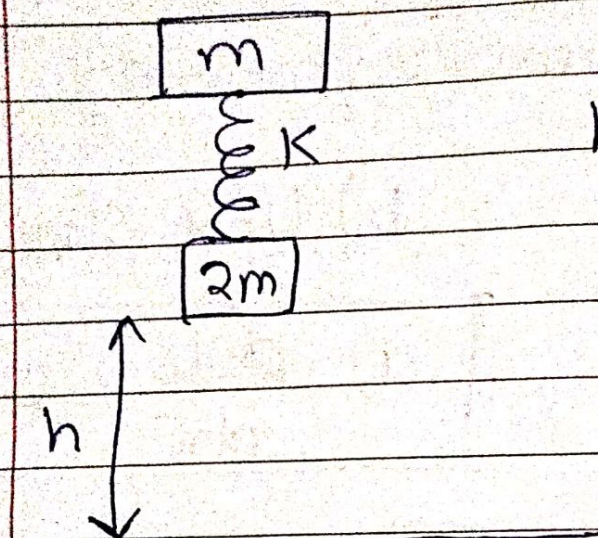
$$W_e + W_{N.C.} = (K_f + U_f) - (K_i + U_i)$$

$\Rightarrow$ When external force $= 0$
Non-conservative force $= 0$

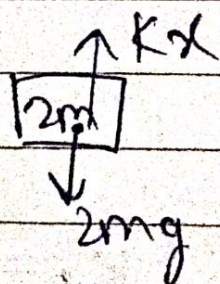
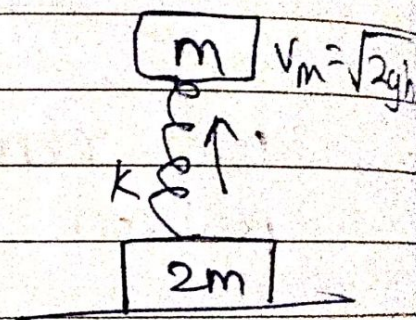
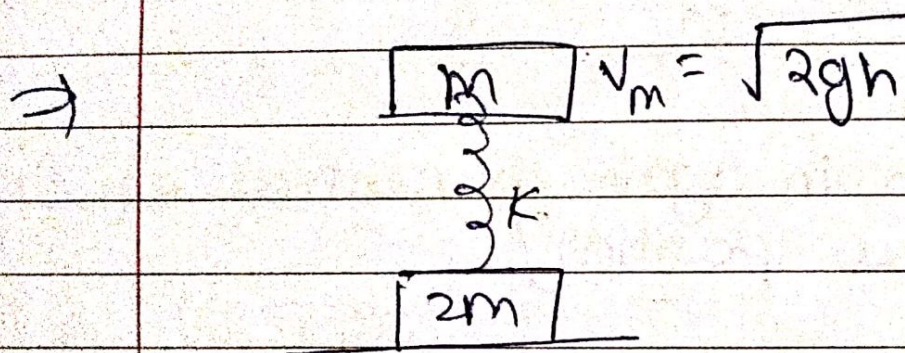
then, $K_i + U_i = K_f + U_f$

Ps
ADV

Que

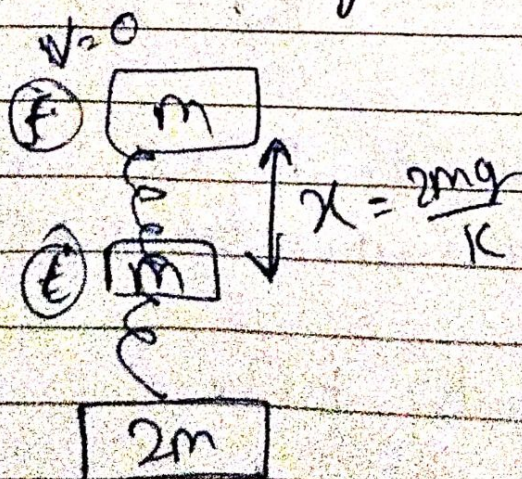


Find the value of h such that $2m$ just manages to rise above ground after collision.



Kx should just more $\Rightarrow$ than $2mg$

$\Rightarrow$ We can take $Kx = 2mg$
 $x = \frac{2mg}{K}$



Using Conservation of mechanical Energy

$$K_f + U_f = K_i + U_i$$

$$0 + mgx + \frac{1}{2}kx^2 = \frac{1}{2}m \cdot 2gh + 0$$

$$\Rightarrow mgx + \frac{1}{2}kx^2 = \frac{1 \cdot 2mgh}{2}$$

$$\Rightarrow \frac{2mgx + kx^2}{2} = mgh$$

$$\Rightarrow 2mgx + kx^2 = 2mgh$$

$$\Rightarrow kx^2 + 2mgx - 2mgh = 0$$

$$x = \frac{2mg}{k}$$

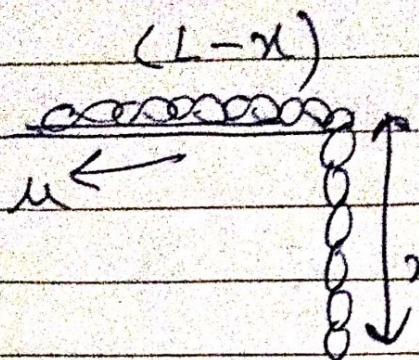
$$\Rightarrow k \cdot \frac{4m^2g^2}{k^2} + 2mg \frac{2mg}{k} - 2mgh = 0$$

$$\Rightarrow \frac{4m^2g^2}{k} + \frac{4m^2g^2}{k} = 2mgh$$

$$\Rightarrow \frac{8m^2g^2}{2k} = mgh$$

$$\Rightarrow \frac{4m^2g^2}{k} = mgh \Rightarrow \boxed{h = \frac{4mg}{k}}$$

Que



A chain of length ' L ' and mass ' M ' is made to sustain on the table find the maximum length it can hang so that body doesn't fall.

$\Rightarrow$ Let the chain to be hanged be of length x .

$\therefore$ chain on table $= (L-x)$

If the L length of chain weighs M
 x length of chain weighs $\frac{Mx}{L}$

As $(L-x)$ length of chain weight $\frac{M(L-x)}{L}$

Now frictional force $= \mu N$

$$\Rightarrow \mu (mg)$$

$$\Rightarrow \mu \frac{M(L-x)}{L} g$$

Force acting on falling chain $= \frac{Mx}{L} g$

→ A body has to stay

$$\therefore \frac{\mu M(L-x)g}{\cancel{L}} = \frac{\cancel{M}xg}{\cancel{L}}$$

$$\Rightarrow \mu(L-x) = x$$

$$\Rightarrow \mu L - \mu x = x$$

$$\Rightarrow \mu L = x + \mu x$$

$$\Rightarrow \mu L = x(1+\mu)$$

$$\Rightarrow \boxed{\frac{\mu L}{1+\mu} = x}$$

Ques In the same Que above find the total work done by frictional force if the chain is pulled horizontal down.

→

$\frac{L}{1+\mu} dm$
 dx
 $\frac{\mu L}{1+\mu}$
 $g/L \rightarrow M$
 $dx \rightarrow M dx$
 L

$$f = \mu mg = \mu \frac{M dx}{L} g$$

$$dw = f dx$$

$$\int dw = \int \frac{\mu M dx}{L} g x$$

$$\Rightarrow \int dw = \frac{\mu M g}{L} \int x dx$$

vary x from $\frac{L}{1+\mu}$ to 0.

$$\therefore \int dw = \frac{\mu M g}{L} \int_0^{\frac{L}{1+\mu}} x dx$$

$$\Rightarrow \frac{\mu M g}{L} \left[\frac{x^2}{2} \right]_0^{\frac{L}{1+\mu}}$$

$$= \frac{\mu M g}{2L} \left[\left(\frac{L}{1+\mu} \right)^2 \right]$$

$$\Rightarrow \frac{\mu M g L^2}{2k(1+\mu)^2}$$

$$\Rightarrow \frac{\mu M g L}{2(1+\mu)^2}$$